Hacking in Parallel ///

Are you old enough to buy this?

Zero-Knowledge Age Restriction for GNU Taler

Özgür Kesim

December 29, 2022

FU Berlin

Prolog

Who am I

Özgür Kesim,

- security consultant for 20+ years,
- PhD candidate at FU Berlin,
- member of GNU Taler dev-team.

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What to expect

Deliverable

Present a solution to age restriction and its integration in GNU Taler.

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Show concepts from cryptography by example:

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Non-goals

Rigorous introduction into GNU Taler

Demos

Sponsors

NGI Pointer program of the European Commission

Project *Concrete Contracts* in the *KMU-innovativ* programm

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Chapters

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Introduction

Age Restriction in E-commerce

Youth protection

Broad consensus in society about the necessity to protect minors from harmful content.

Also wanted from policy makers:

> *11. Member states should encourage the **use of conditional access tools** by content and service providers in relation to content harmful to minors, **such as age-verification systems**, ...*

From the *Recommendation Rec (2001) 8 of the Committee of Ministers to member states on self-regulation concerning cyber content* of the Council of Europe.

Age restriction in E-commerce

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Verification of minimum age requirements in e-commerce.

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For age-restriction, the lowest level of authority is:

Parents, guardians and caretakers

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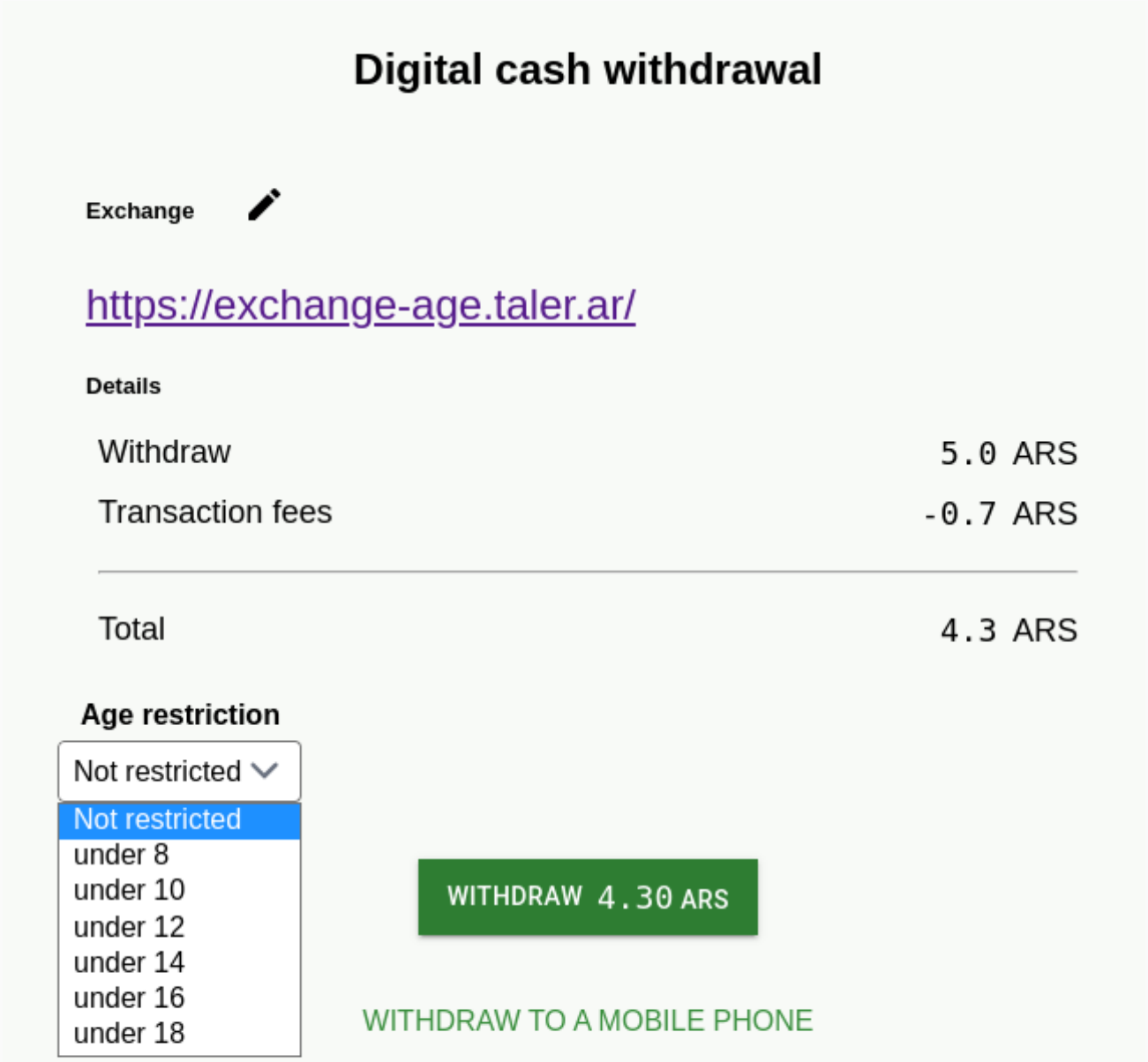
5. is **practical and efficient**.

Teaser


Digital cash withdrawal
Exchange
https://exchange-age.taler.ar/
Details
Withdraw 5.0 ARS
Transaction fees -0.7 ARS
Total 4.3 ARS
Age restriction
Not restricted
Not restricted
under 8
under 10
under 12
under 14
under 16
under 18
WITHDRAW 4.30 ARS
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The quest for a solution to age restriction

A journey through cryptic territory

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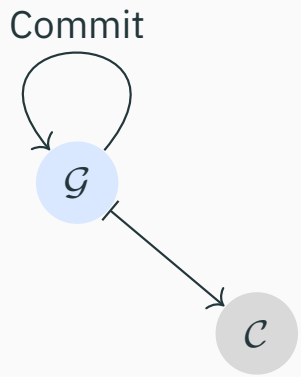

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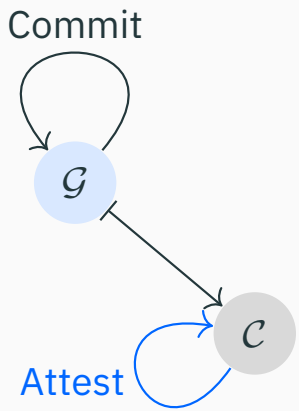

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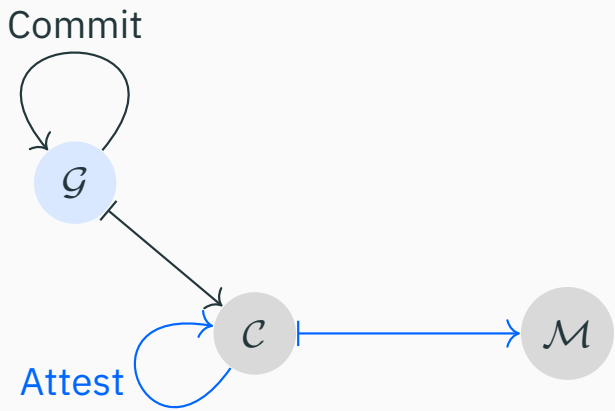

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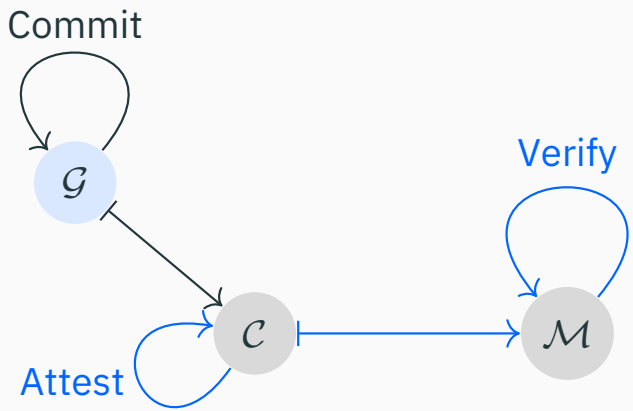

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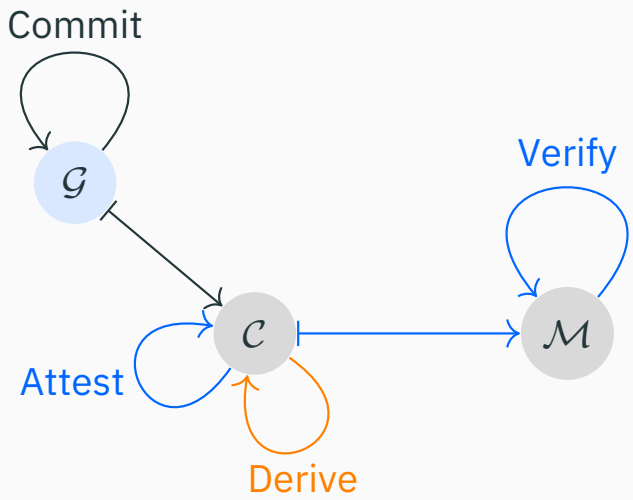

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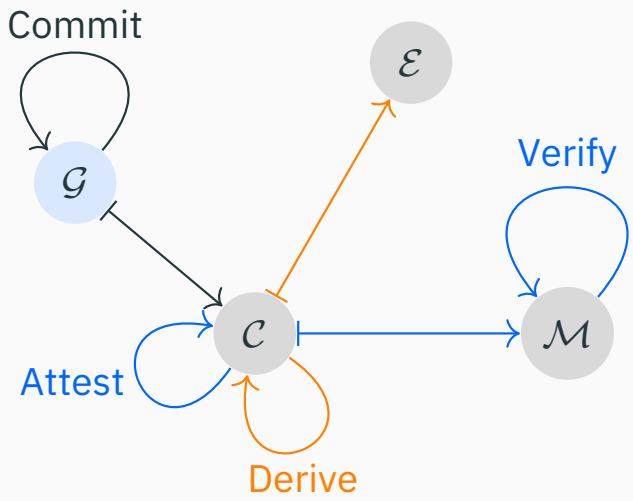

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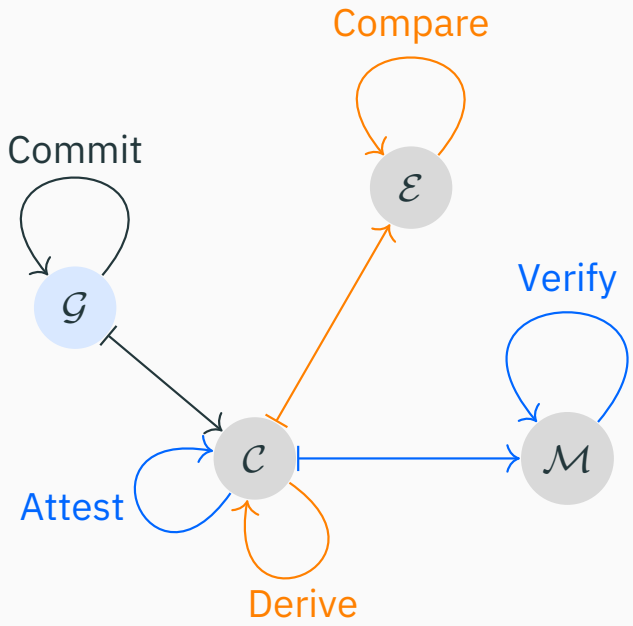

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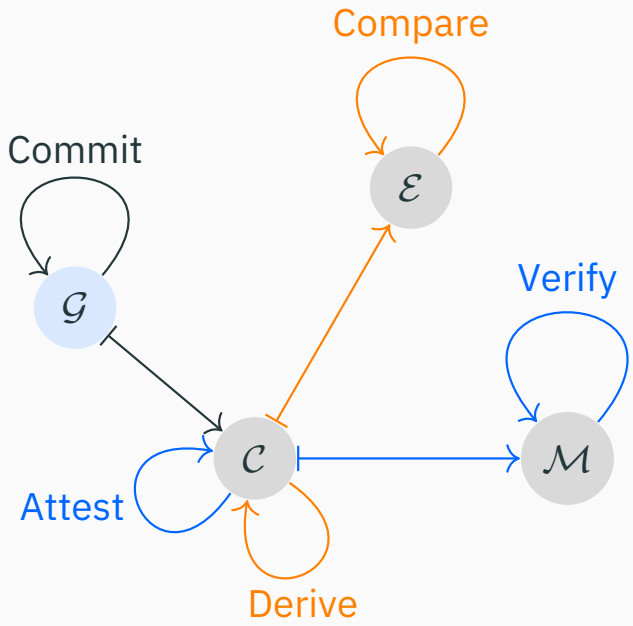

Specification of the Function Signatures

Searching for functions

- Commit
- Attest
- Verify
- Derive
- Compare

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with $\Omega, \mathbb{P}, \mathbb{O}, \mathbb{T}, \mathbb{B}$ sufficiently large sets.

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We will define basic and security requirements later.

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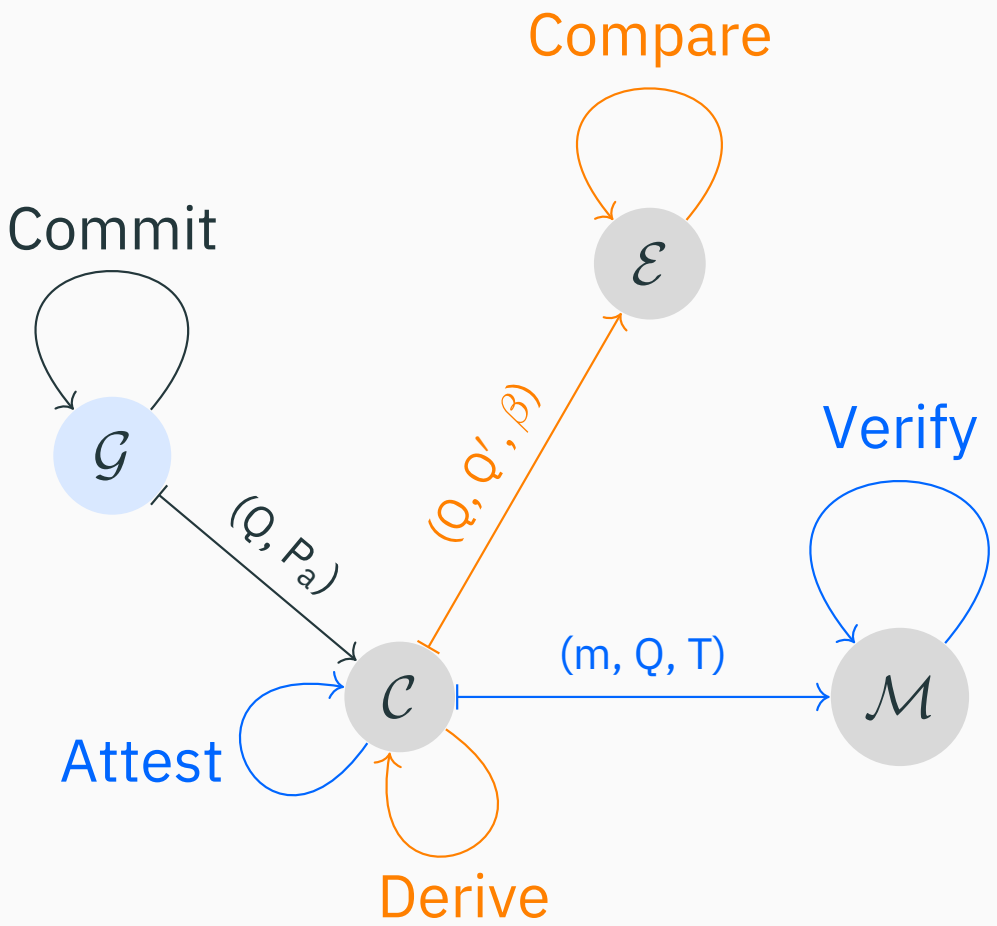
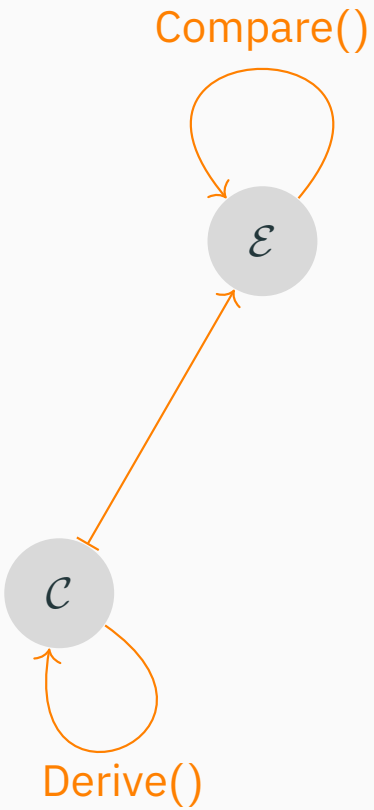
Naïve scheme



Problem of unlinkability



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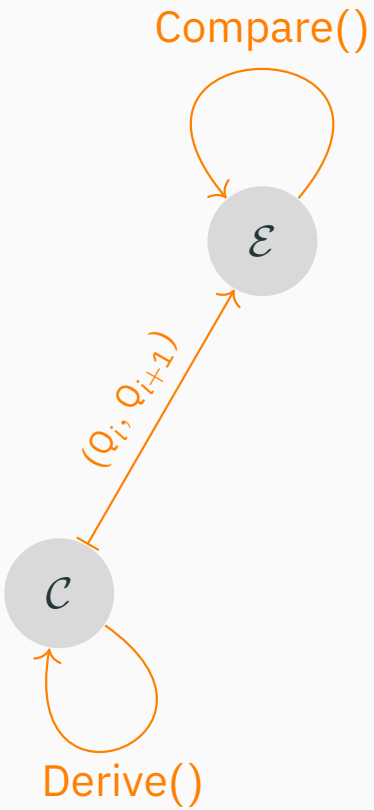

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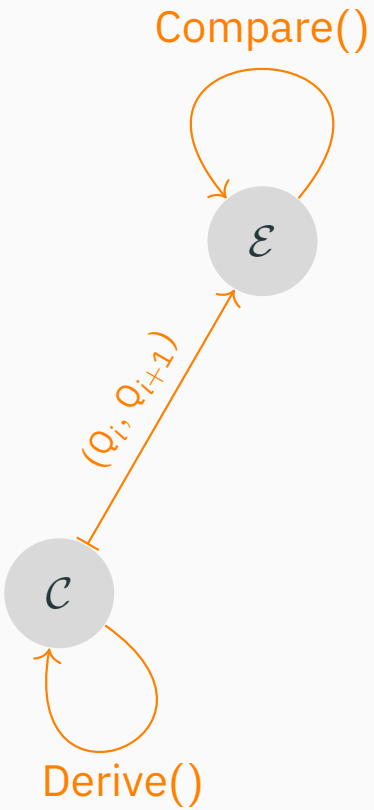

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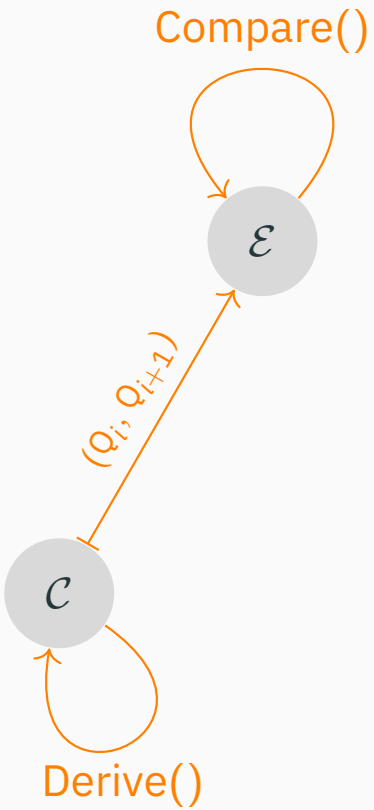

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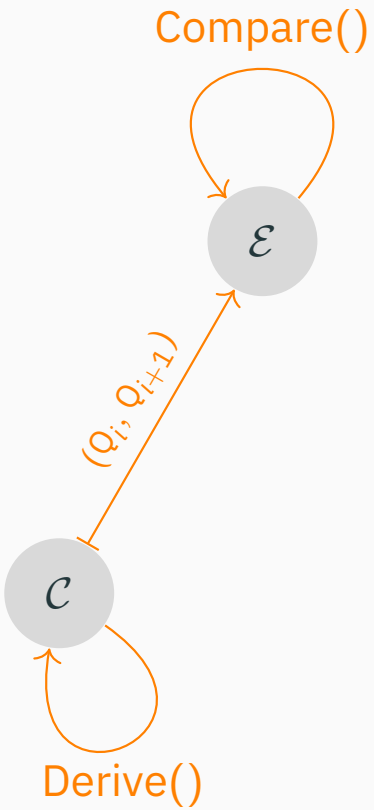

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$\Longrightarrow$ **Unlinkability broken**

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If all steps succeed, Q_γ is the new commitment.

Achieving Unlinkability

With DeriveCompare$_\kappa$

- $\mathcal{E}$ learns nothing about Q_γ or $H(\mathsf{Q}_\gamma)$,
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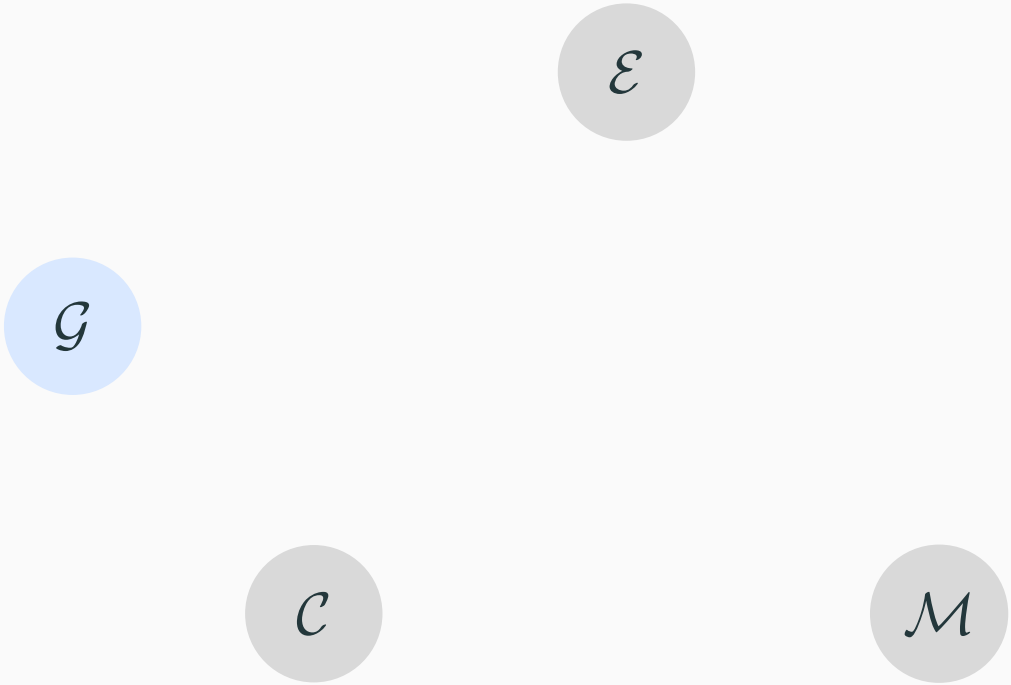
Notes:

- similar to the cut&choose *refresh* protocol in GNU Taler
- still need to define Derive() and Compare().

Refined scheme



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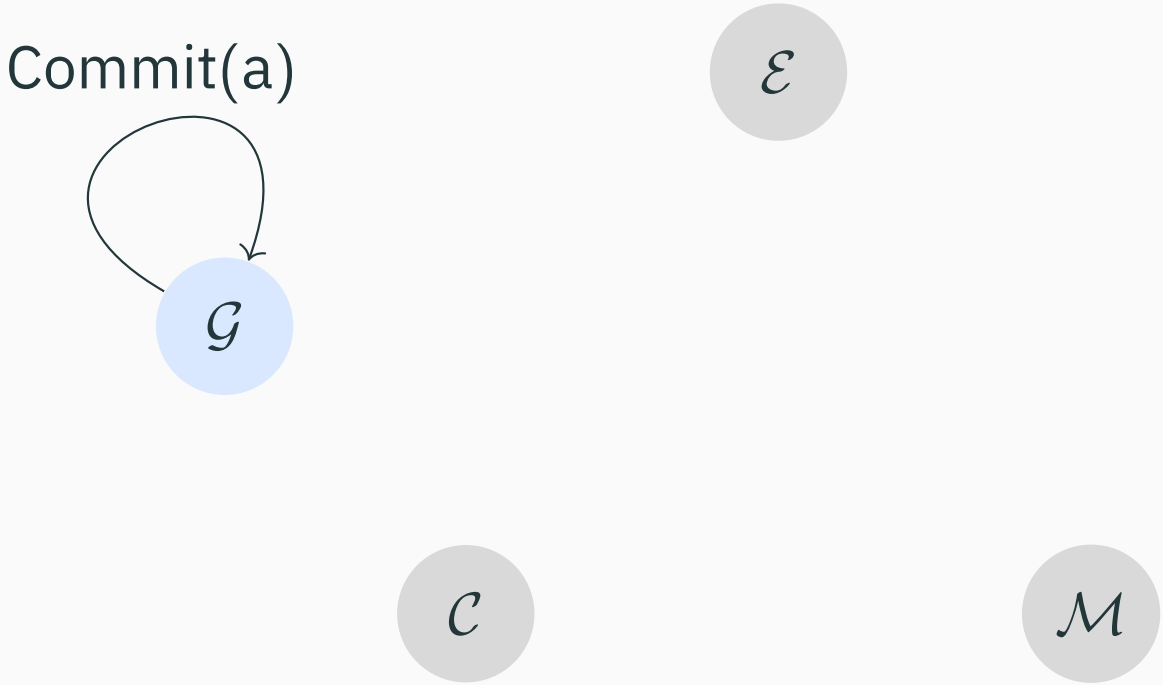

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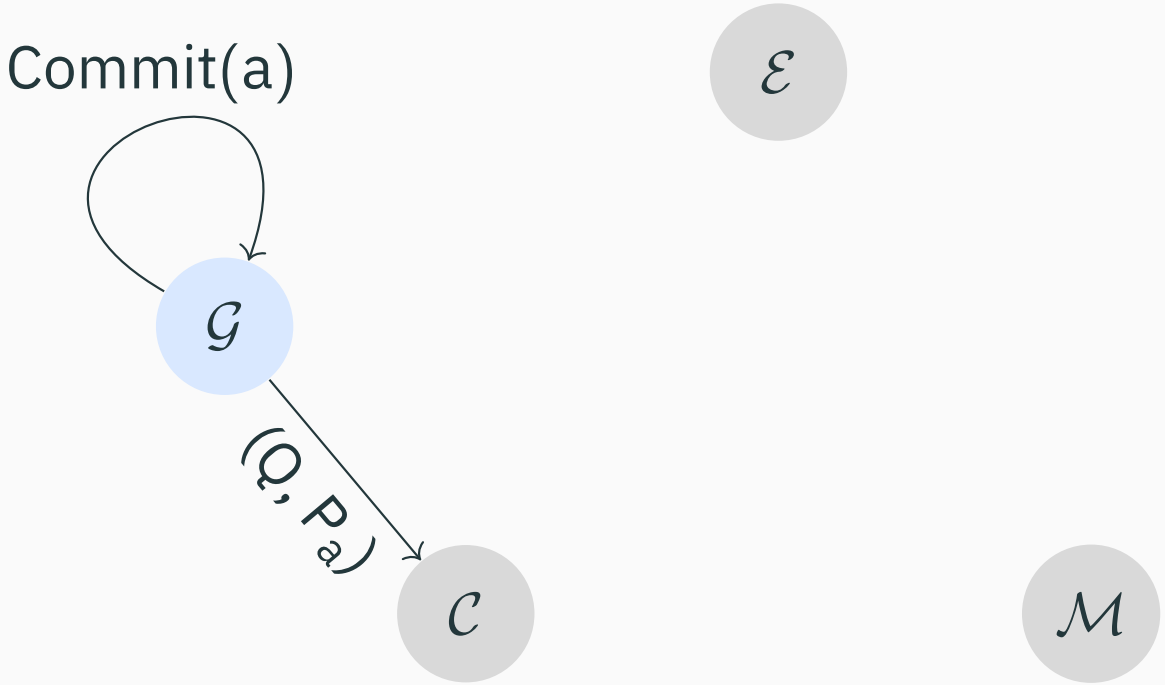

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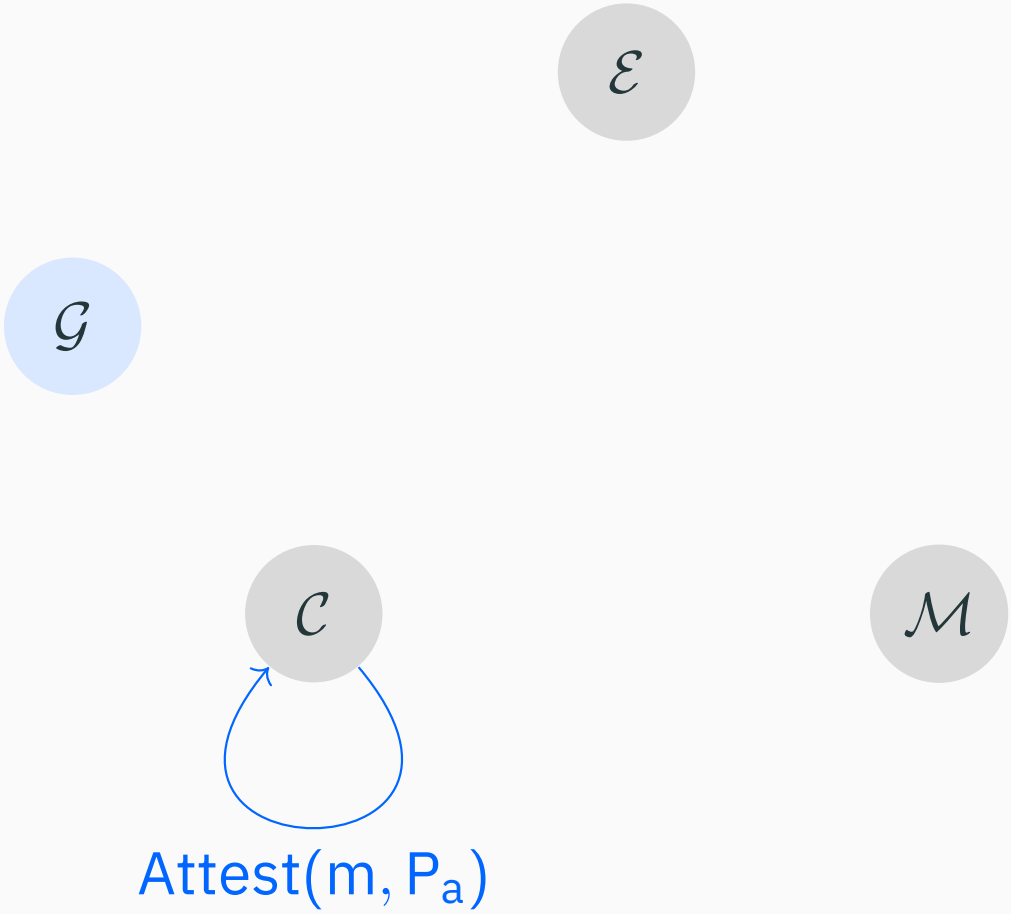

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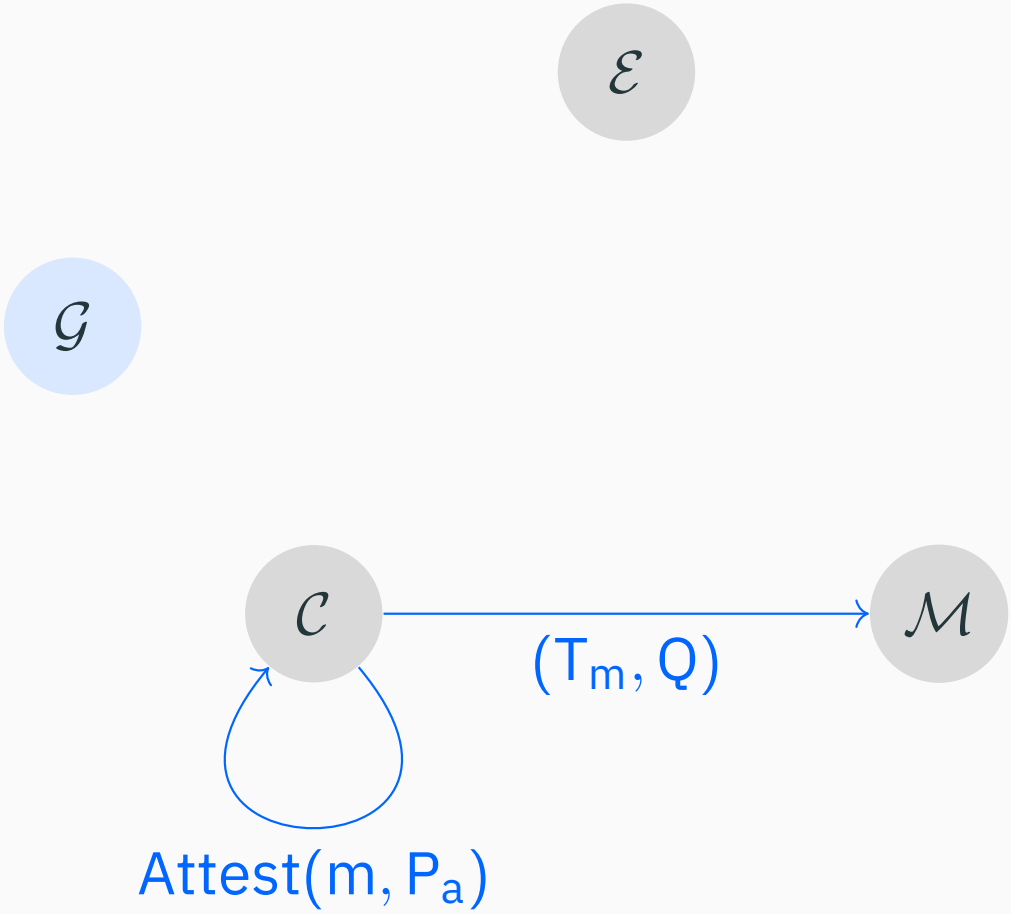

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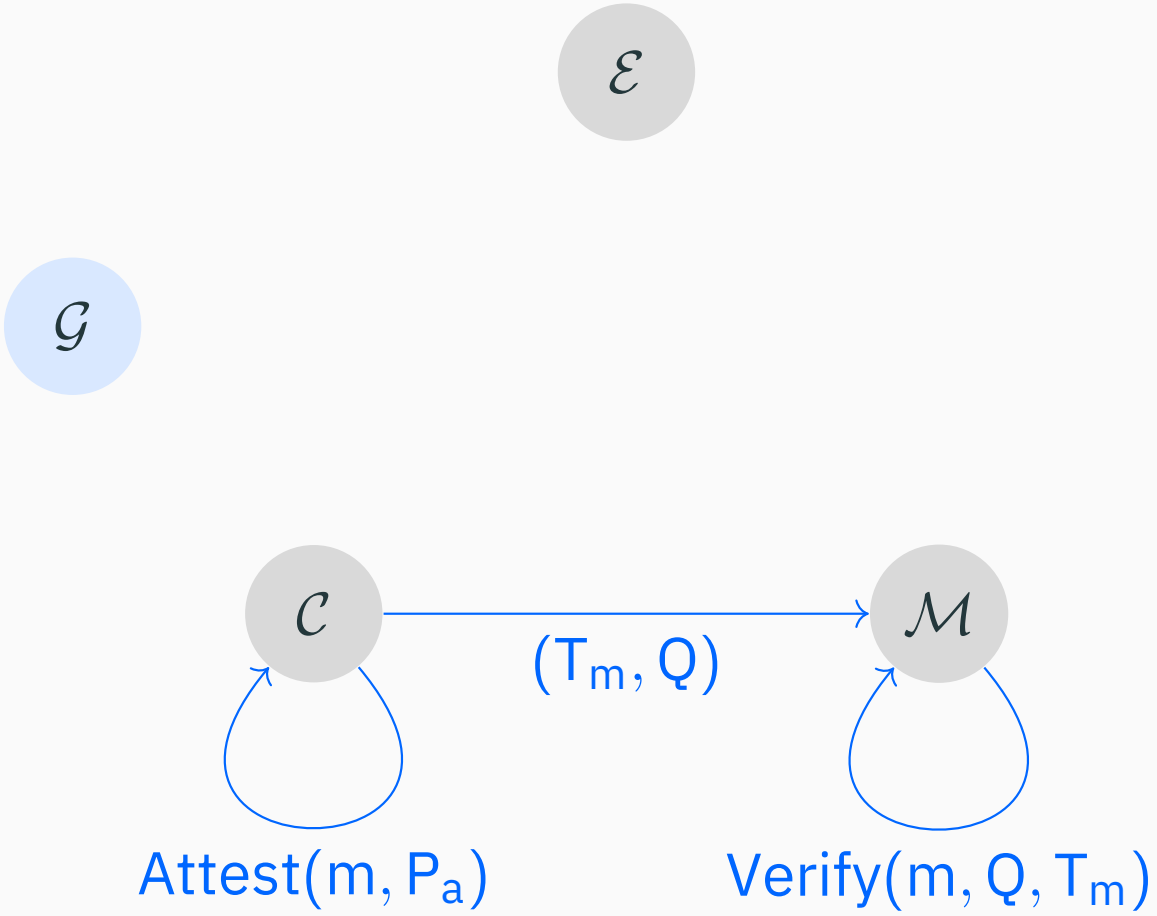

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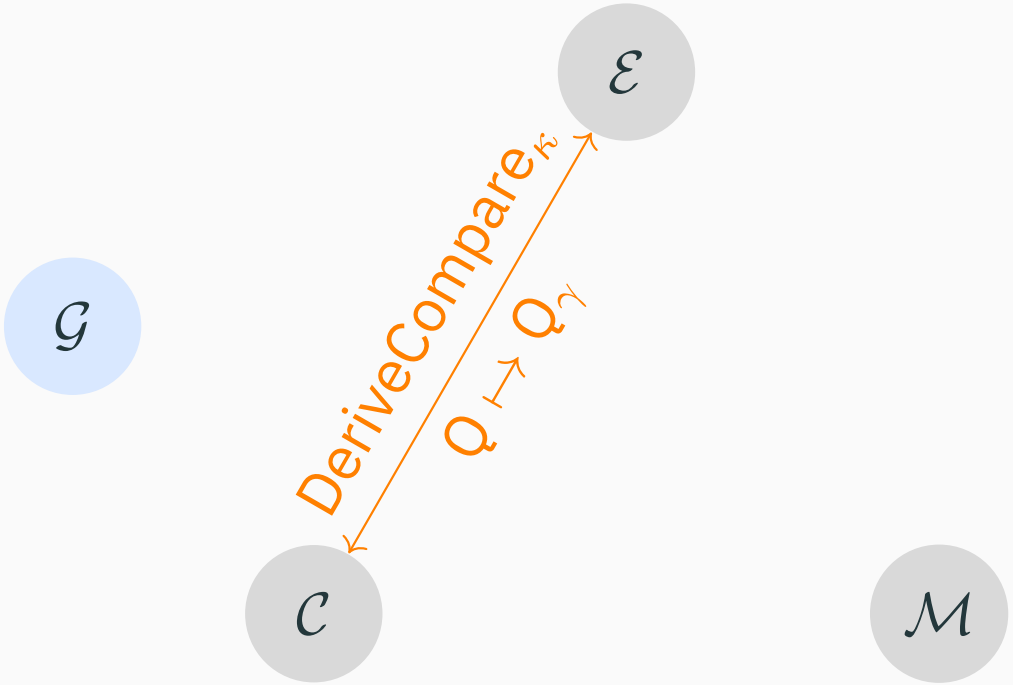

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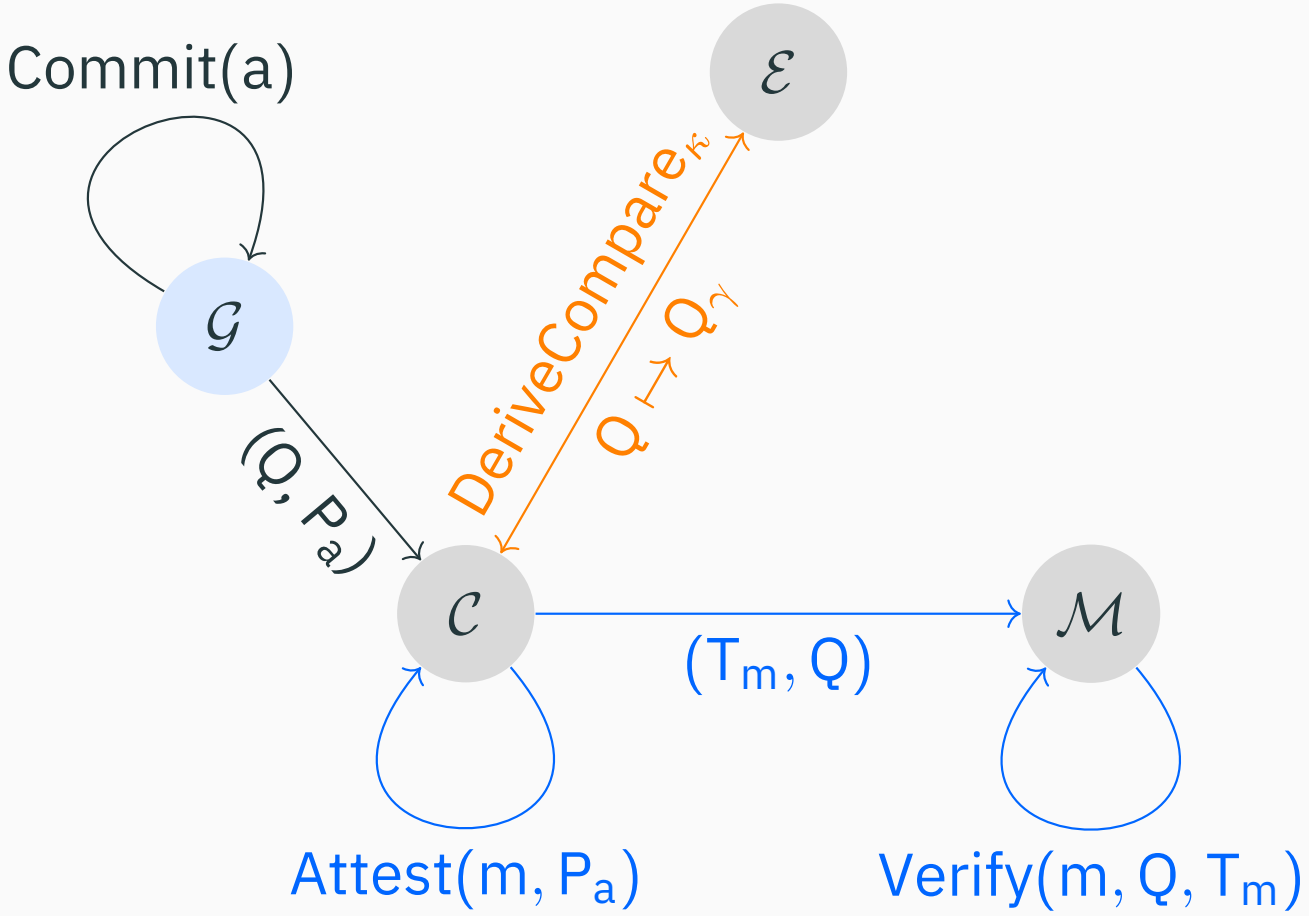

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We need more requirements.

Requirements

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Candidate functions

$$(\mathsf{Commit}, \mathsf{Attest}, \mathsf{Verify}, \mathsf{Derive}, \mathsf{Compare})$$

must meet *basic requirements*:

- Existence of attestations
- Efficacy of attestations
- Derivability of commitments and attestations

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More details in the published paper and Appendix.

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Adversaries are arbitrary polynomial-time algorithms, acting on all relevant input.

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Requirement: Unforgeability of minimum age

$$\bigvee_{\mathcal{A} \in \mathfrak{A}(\mathbb{N}_{\mathsf{M}} \times \mathbb{O} \times \mathbb{P} \to \mathbb{N}_{\mathsf{M}} \times \mathbb{T})} : \Pr\left[G_{\mathcal{A}}^{\mathsf{FA}} = 1\right] \leq \epsilon$$

Our task

Finding functions

(Commit, Attest, Verify, Derive, Compare)

that meet the basic and security requirements.

A solution

Instantiation with ECDSA

We propose a solution based on ECDSA.

Think: One key-pair per age group.

Definition of Commit with ECDSA

To Commit to age group $a \in \{1, \ldots, M\}$

1. Guardian generates ECDSA-keypairs, one per age group:

$$\langle (q_1, p_1), \ldots, (q_M, p_M) \rangle$$

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$$\textbf{Commitment:}\quad \vec{\mathsf{Q}} := (q_1, \ \ldots \ \ldots \ \ldots \ , q_\mathsf{M})$$
$$\textbf{Proof:}\quad \vec{\mathsf{P}}_\mathsf{a} := (p_1, \ldots, p_\mathsf{a}, \bot, \ldots, \bot)$$

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3. Guardian gives child $\langle \vec{Q}, \vec{P}_a \rangle$

Attest and Verify with ECDSA

Child has

- ordered public-keys $\vec{\mathsf{Q}} = (q_1, \ldots \ \ldots \ \ldots, q_{\mathsf{M}})$,
- (some) private-keys $\vec{\mathsf{P}} = (p_1, \ldots, p_{\mathsf{a}}, \perp, \ldots, \perp)$.

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Sign a message with ECDSA using private key p_{m}. The signature σ_{m} is the attestation.

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To Verify a minimum age (group) m**:**

Verify the ECDSA-Signature σ_{m} with public key q_{m}.

Derive and Compare with ECDSA

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To Derive new $\vec{Q}'$ and $\vec{P}'$: Choose random $\beta \in \mathbb{Z}_g$ and calculate

$$\vec{Q}' = (q'_1, \ \ldots \ \ldots \ \ldots, q'_M) \quad := (\beta * q_1, \ldots \ \ldots, \beta * q_M),$$
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Exchange gets $\vec{Q} = (q_1, \ldots, q_M)$, $\vec{Q}' = (q'_1, \ldots, q'_M)$ and β

To Compare, calculate: $(\beta * q_1, \ldots, \beta * q_M) \stackrel{?}{=} (q'_1, \ldots, q'_M)$

Instantiation with ECDSA

Functions (Commit, Attest, Verify, Derive, Compare) as defined in the instantiation with ECDSA

- meet the basic requirements,
- also meet all security requirements.

Security proofs by reduction, details are in the paper.

Example: Proof of Unforgeability

Proof by reduction:

Game $G^{\mathrm{FA}}_{\mathcal{A}}$: Forging an attest

1. $(\mathsf{a}, \omega) \xleftarrow{\$} \mathbb{N}_{\mathsf{M}-1} \times \Omega$
2. $(\mathsf{Q}, \mathsf{P}) \leftarrow \mathrm{Commit}(\mathsf{a}, \omega)$
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4. Return 0 if $\mathsf{m} \leq \mathsf{a}$
5. Return $\mathrm{Verify}(\mathsf{m}, \mathsf{Q}, \mathsf{T})$

Requirement:

$$\bigvee_{\mathcal{A}} : \Pr\left[G^{\mathrm{FA}}_{\mathcal{A}} = 1\right] \leq \epsilon$$

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$\Longrightarrow$ So winning this game would require to existentially forge the signature, which is negligible.

Instantiation with Edx25519

But... isn’t ECDSA considered to be difficult to implement correctly?

Instantiation with Edx25519

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We also formally define another signature scheme, Edx25519:

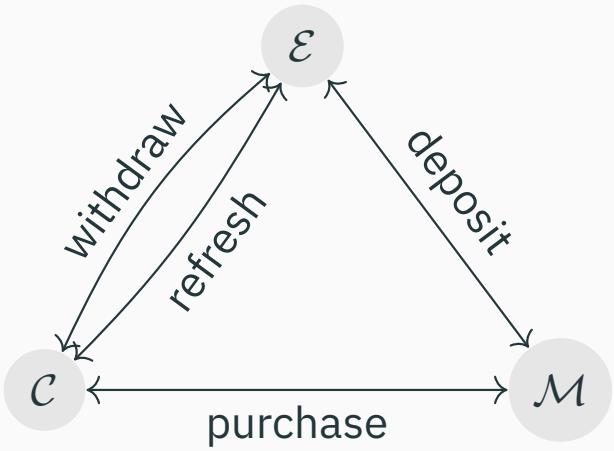
- based on EdDSA (Bernstein et al.),
- generates compatible signatures,
- allows for key derivation from both, private and public keys, independently and
- is already in use in GNUnet.

Current implementation of age restriction in GNU Taler uses Edx25519.

Integration with GNU Taler

GNU Taler

https://www.taler.net



- Protocol suite for online payment services
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- Privacy preserving: anonymous and unlinkable payments

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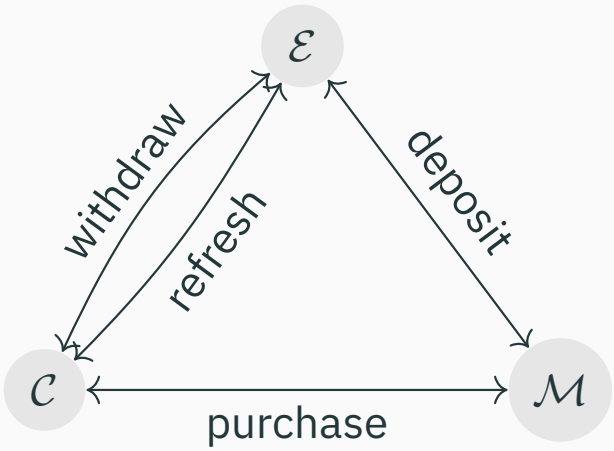

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- Coins are public-/private key-pairs (C_p, c_s).
- Exchange blindly signs $H(C_p)$ with denomination key d_p
- Verification:

$$1 \stackrel{?}{=} \text{SigCheck}\left(H(C_p), D_p, \sigma_p\right)$$

(D_p = public key of denomination and σ_p = signature)

Integration with GNU Taler

Binding age restriction to coins

To bind an age commitment Q to a coin C_p, instead of blindly signing

$$H(C_p),$$

$\mathcal{E}$ now blindly signs

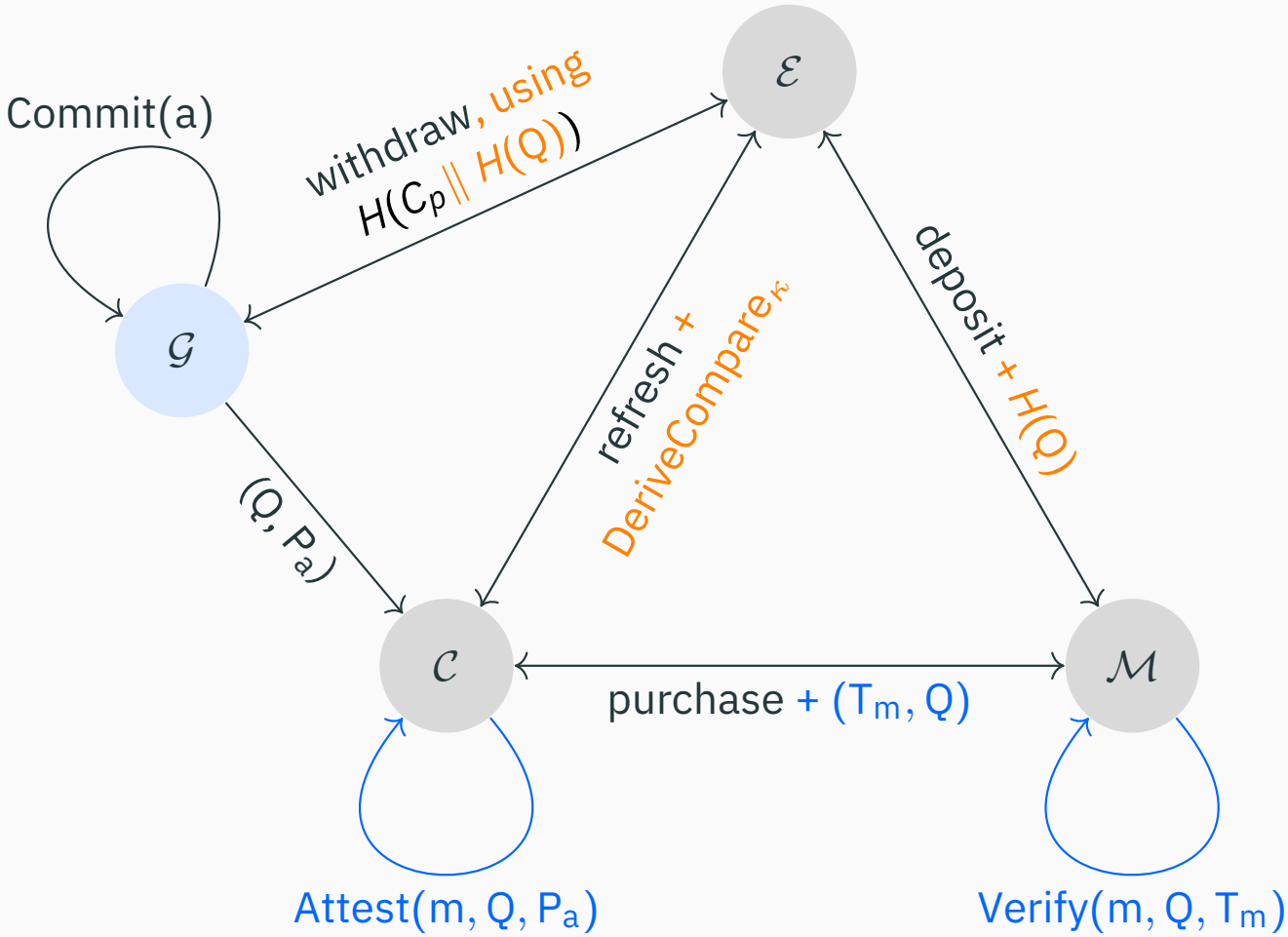
$$H\left(C_p \parallel H(\mathrm{Q})\right)$$

Therefore, verfication of a coin now requires $H(\mathrm{Q})$, too:

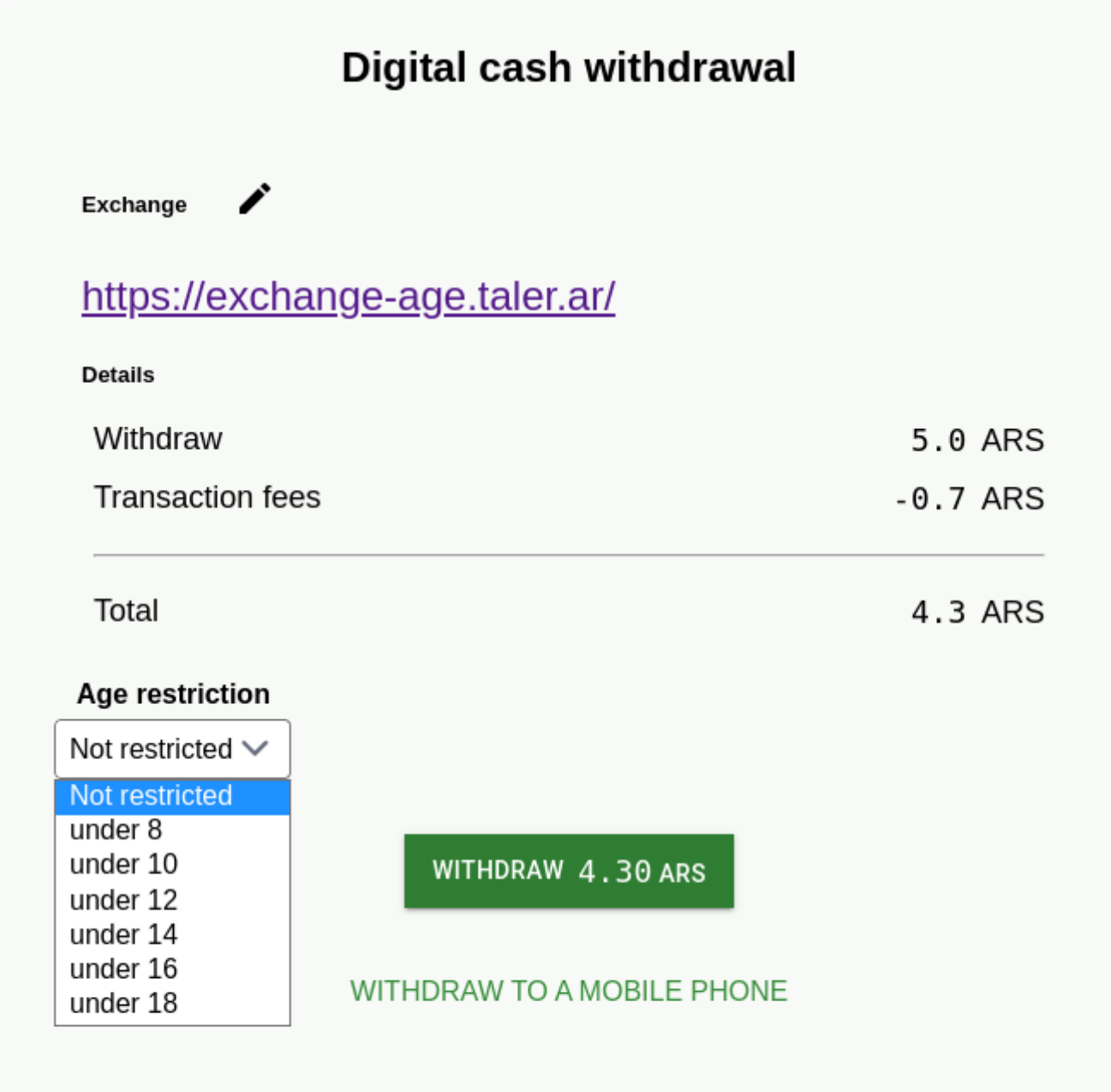
$$1 \stackrel{?}{=} \mathrm{SigCheck}\left(H\left(C_p \parallel H(\mathrm{Q})\right), D_p, \sigma_p\right)$$

Integration with GNU Taler

Integrated schemes



Age restriction in the wallet



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taler

Interested in GNU Taler?

We are looking for developers, testers, users!

Intro: `https://taler.net`

Learn: `https://docs.taler.net`

Develop: `https://git.taler.net`, `https://bugs.taler.net`

Discussion & Conclusion

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Our scheme offers a solution that

- aligns with subsidiarity
- preserves privacy
- is efficient
- and an alternative to IMS

Thank you!

Questions?

oec-taler@kesim.org
@oec@mathstodon.xyz

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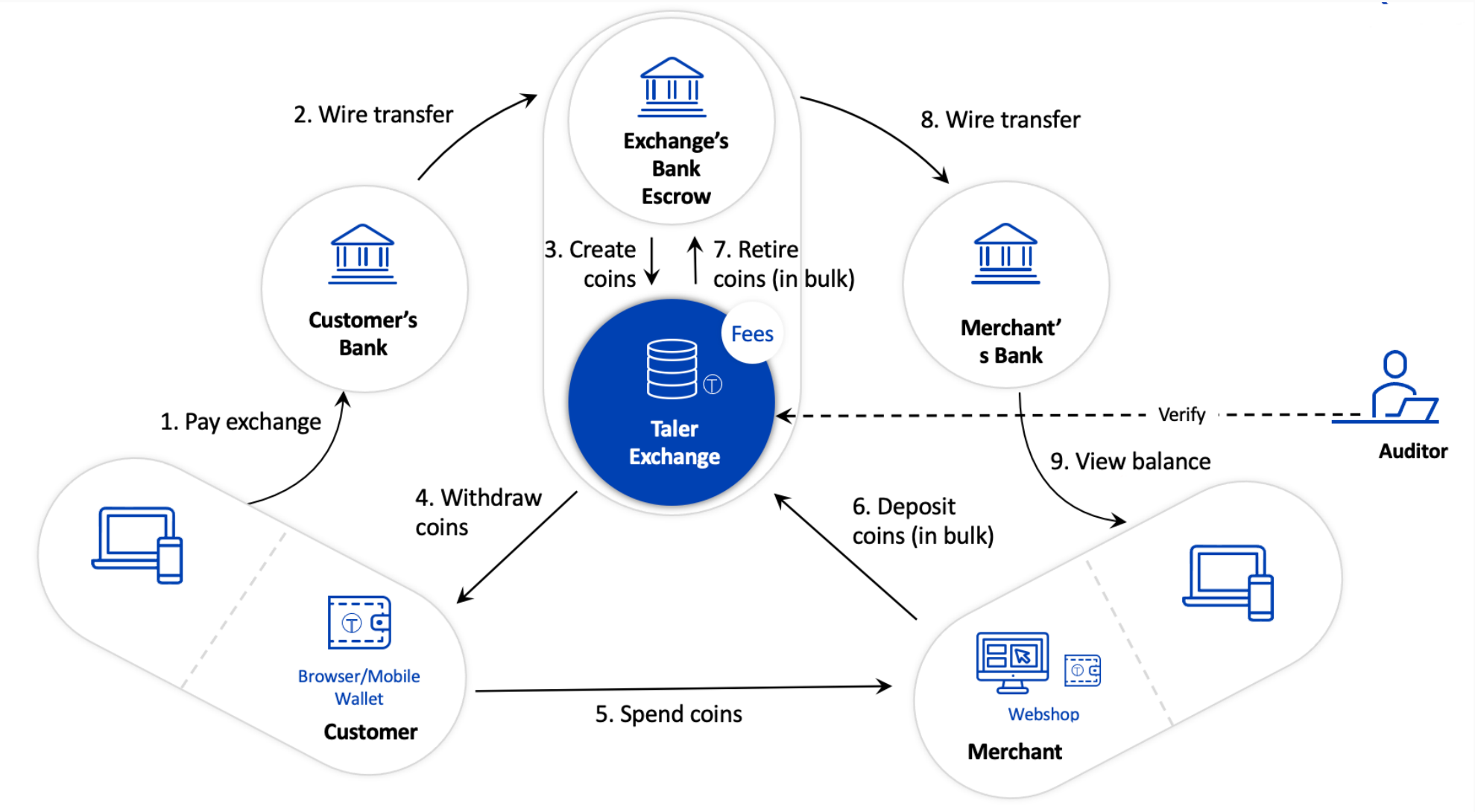
Taler Overview



Basic Requirements - Details

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Derivability of commitments and attestations ...

More details in the published paper.

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B:
- signs m', by calculating $\sigma' := (m')^d \mod N$ *(B doesn't learn m)*

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- signs m', by calculating $\sigma' := (m')^d \mod N$ *(B doesn't learn m)*
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back to *taler* or *binding*

Reminder: RSA blind signature

In RSA, a public key (e, N) and private key (d, N) have the property

$$x^{ed} = x \mod N$$

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$\Longrightarrow$ σ is a valid RSA signature to message m.

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